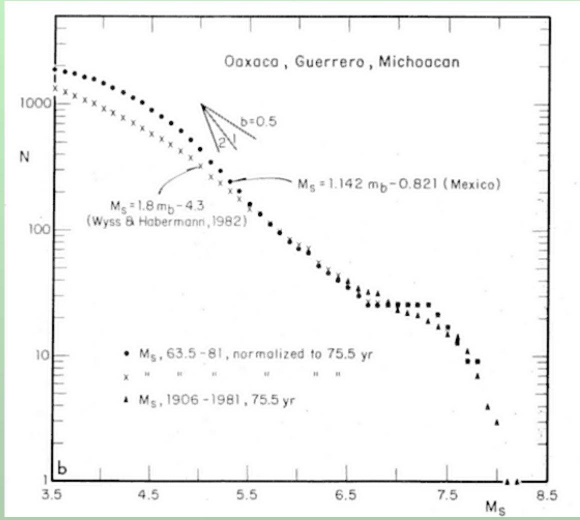
Evidences of complexity of magnitude distribution, obtained from a non-parametric testing procedure

Stanislaw Lasocki

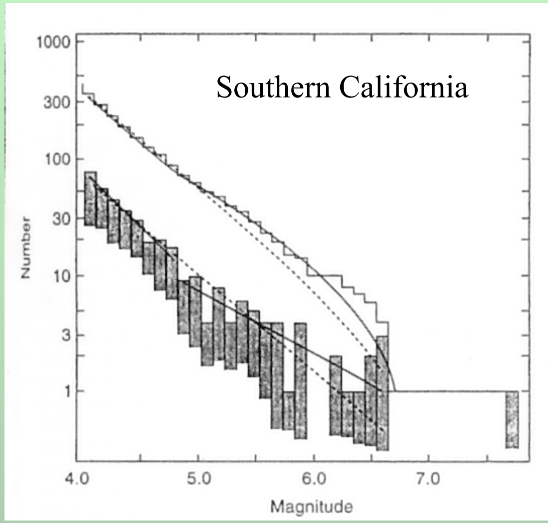
lasocki@geol.agh.edu.pl

AGH University of Science and Technology, Faculty of Geology, Geophysics and Environmental Protection
Al. Mickiewicza 30, 30-059 Krakow, Poland

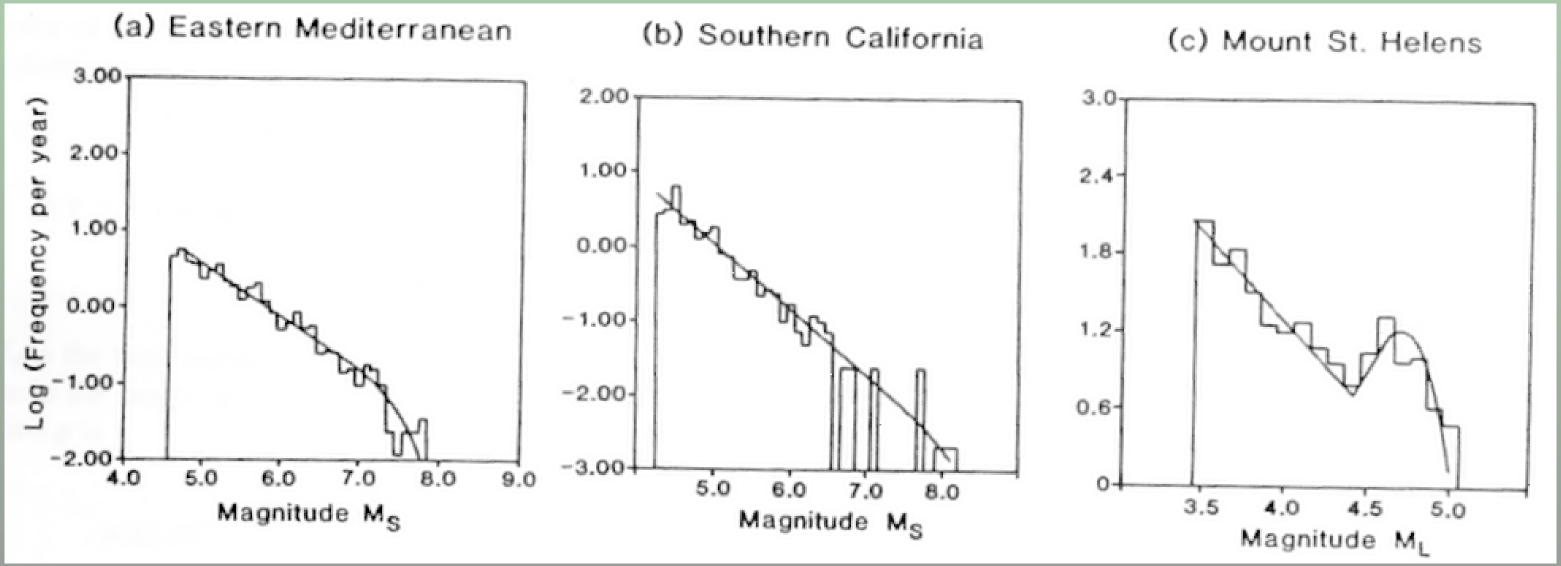
Complex magnitude distributions



Singh, et. al., 1983, BSSA 73, 1779-1796



Knopoff, 2000, PNAS 97, 11880-11884



Main, 1995, BSSA 85, 1299-1308

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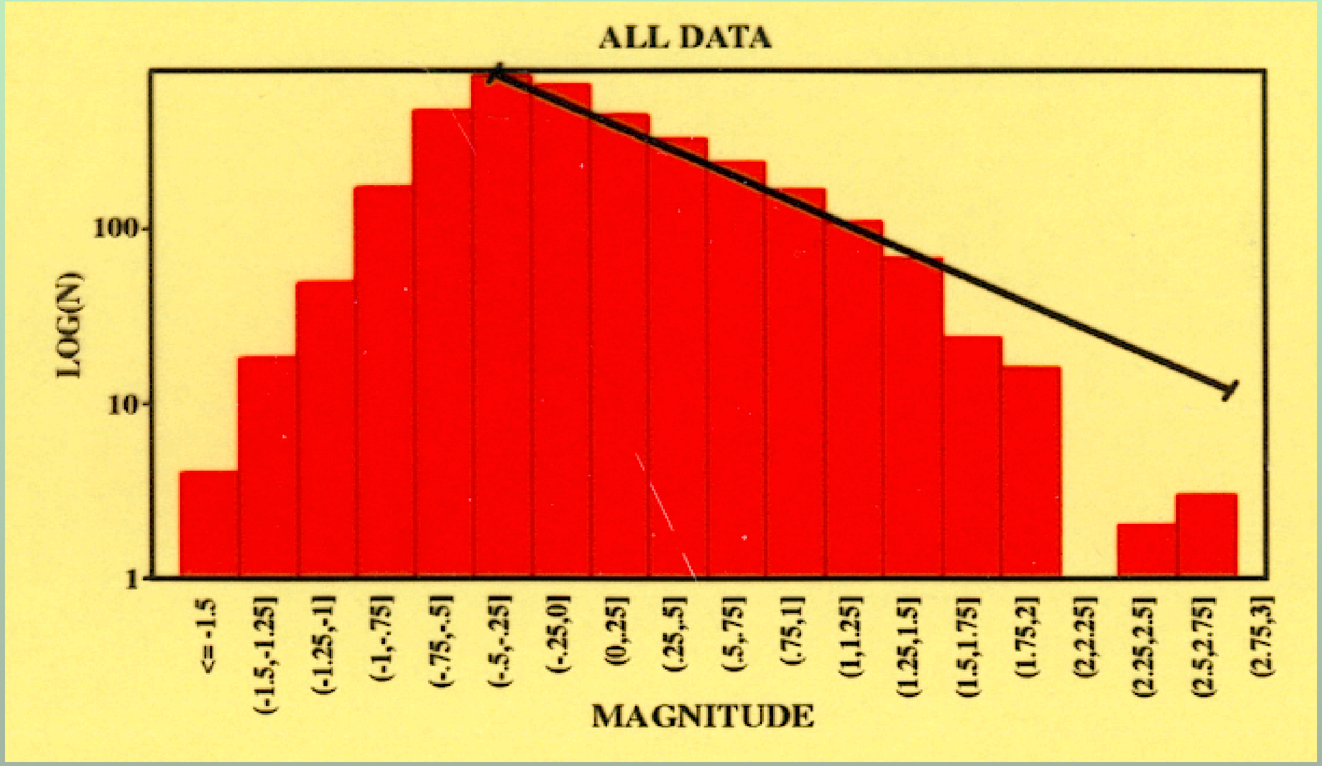
$f_{lin}(M)$: exponential (G-R)

$f^{*}_{smooth}(M)$: tapered G-R (Jackson, Kagan, 1999)
gamma for M_0 (Kagan, 1999)
generalized Pareto for M_0 (Pisarenko, Sornette, 2003)
Weibull (Lasocki, 1993)
double exponential (Lomnitz-Adler, Lomnitz, 1978)
normal (Niazi,1964)
Utsu (1971), Makjanić (1972), Saito et al. (1973), Purcaru (1975), Seino et al. (1989) ...

2. Exponential-like shape of $f_{act}(M)$



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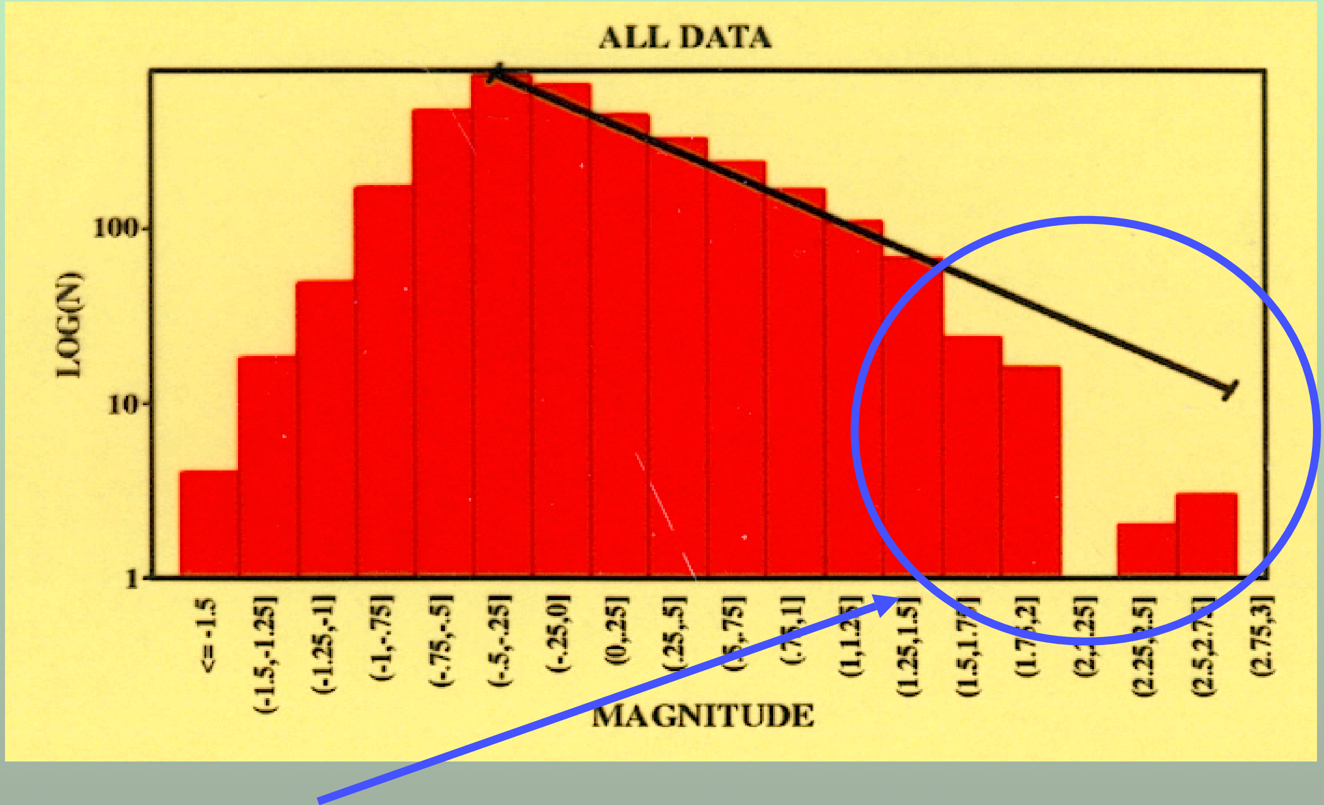

???

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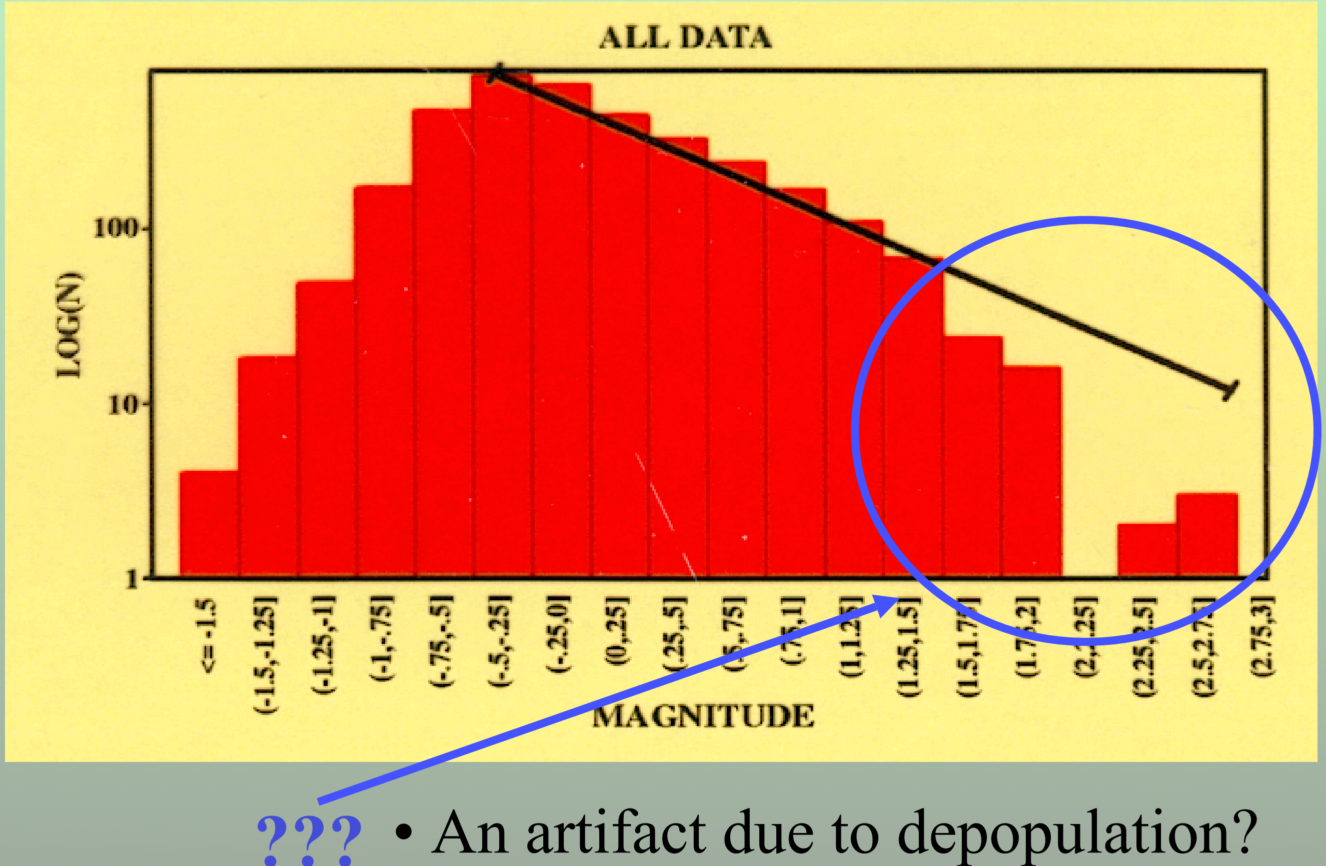

• An artifact due to depopulation?

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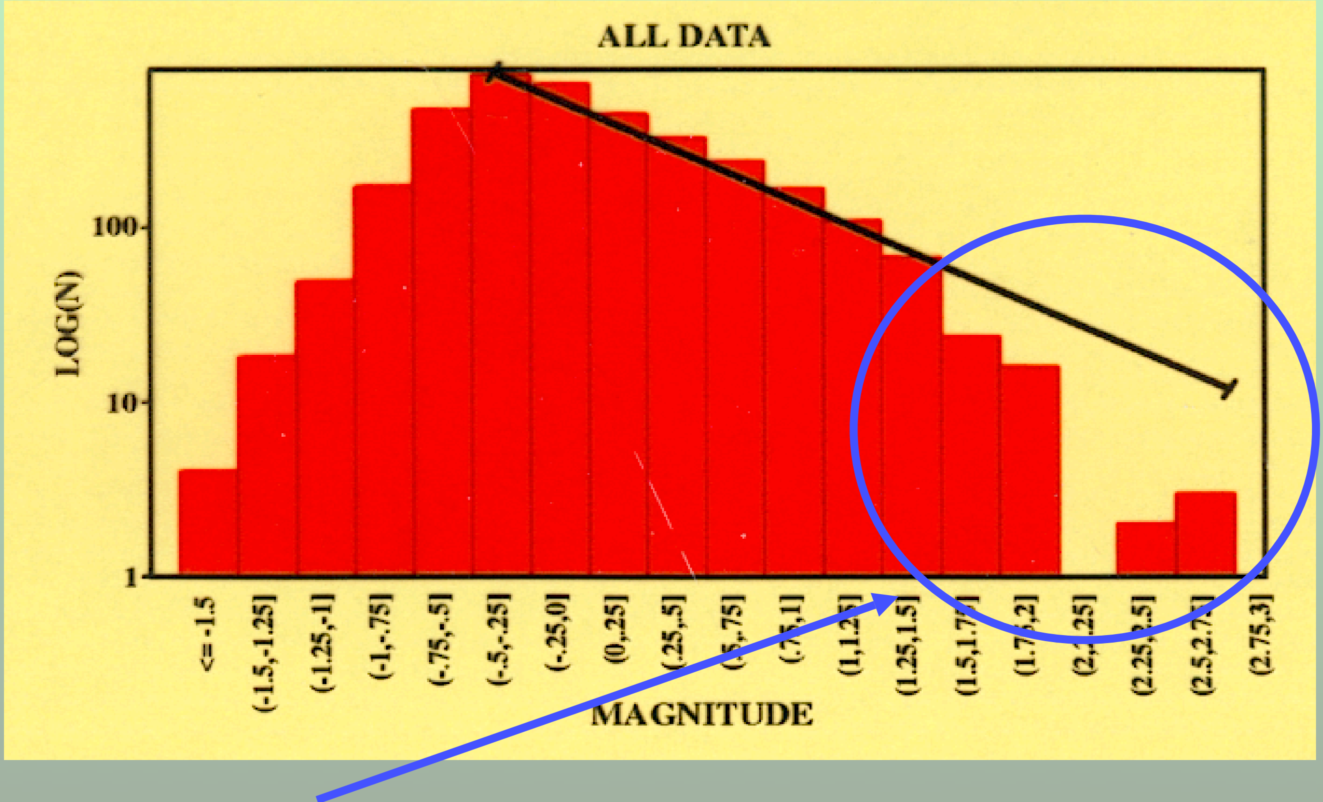

???
- An artifact due to depopulation?
- A statistical scatter?

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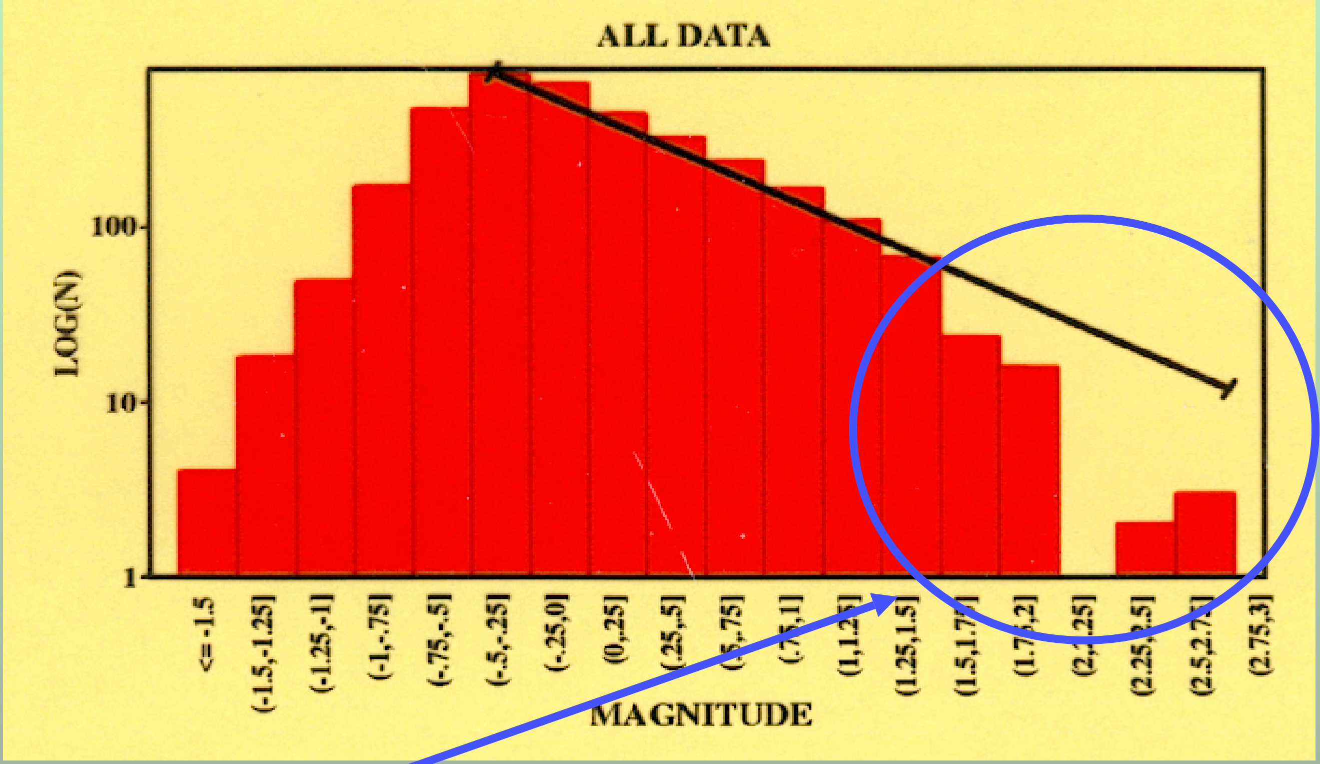

??? • An artifact due to depopulation?

• A statistical scatter?

• A real break in scaling law?

Model-free testing: The smoothed bootstrap test for multimodality *(Silverman,1986; Efron, Tibshirani, 1998)*:

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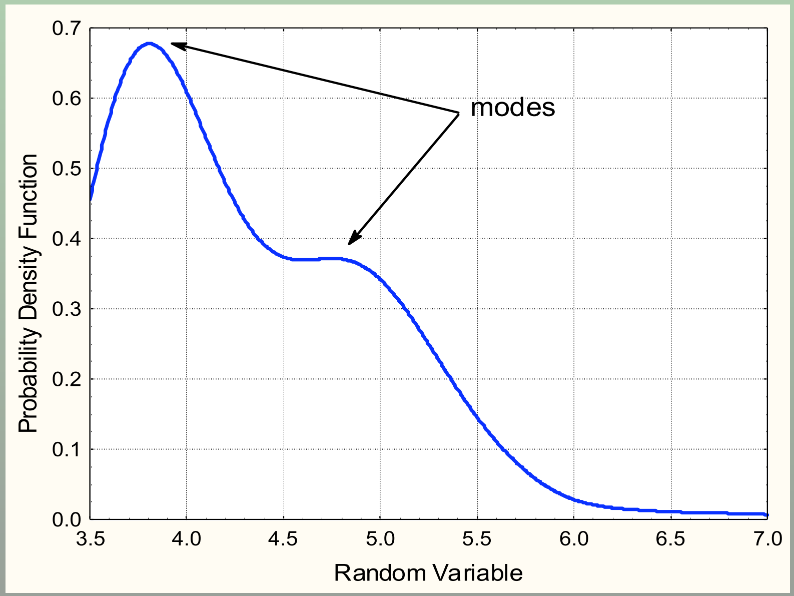
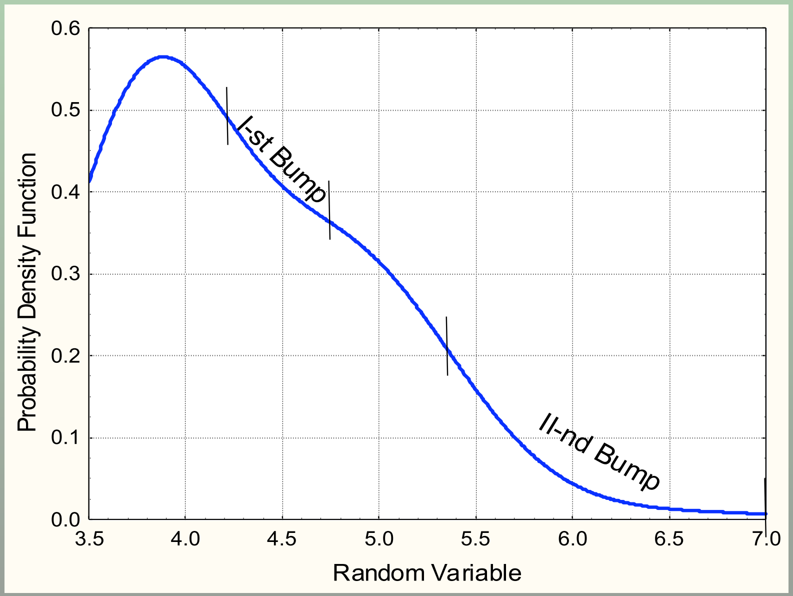
$\boxed{H_0^1\left(f_{act}(M)\, is\ unimodal\right)}$ $\boxed{H_0^2\left(f_{act}(M)\, is\ unibumpal\right)}$

Mode = a local maximum of PDF

Bump = [a,b]: PDF concave over [a,b] and not over any larger interval





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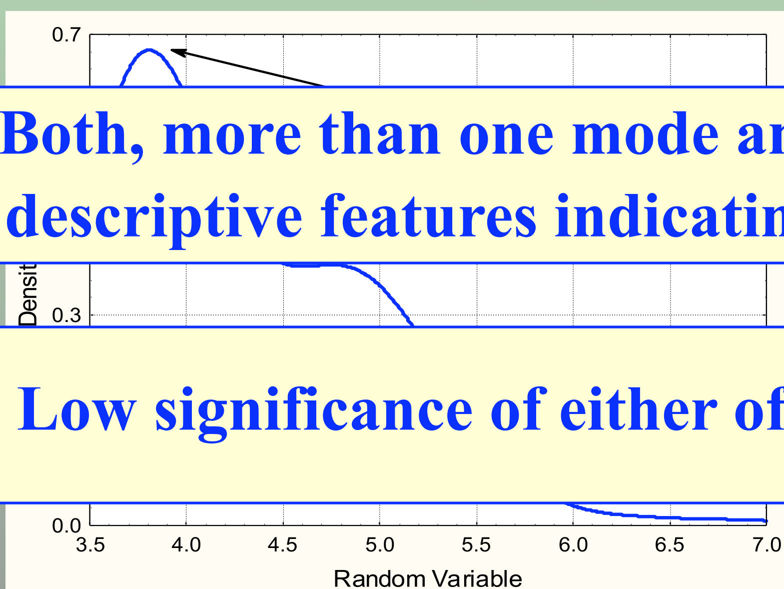

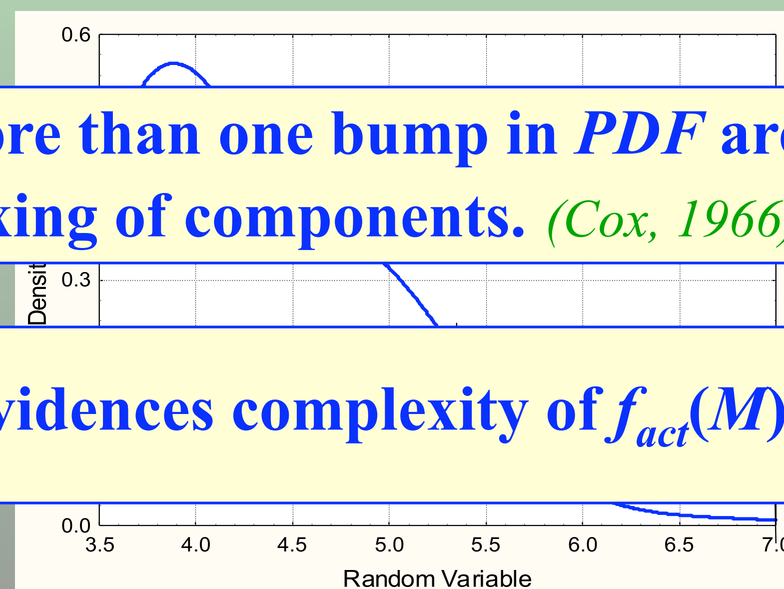

Both, more than one mode and more than one bump in *PDF* are descriptive features indicating mixing of components. *(Cox, 1966)*

Low significance of either of H_0 evidences complexity of $f_{act}(M)$

TESTING PROCEDURE SCHEME

EQ DATA

RANDOMIZATION

TESTING FOR MULTIMODALITY

EVALUATING CRITICAL SMOOTHING FACTOR

SMOOTH BOOTSTRAPPING

ESTIMATING H_0 SIGNIFICANCE

TEST CALIBRATION

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DATA RANDOMIZATION

To avoid a spurious complexity of $f_{act}(M)$ due to repetitions the observed magnitudes M_{obs} are exponentially randomized within their round-off intervals δM (=0.1)

$$M_{obs} \text{ ® } M_{rand}$$

$$M_{rand} = F_e^{-1}\{u[F_e(M_{obs} + 0.5\delta M) - F_e(M_{obs} - 0.5\delta M)] + F_e(M_{obs} - 0.5\delta M)\}$$

$$F_e(M) = 1 - \exp[-\beta(M - M_c)]$$

$$u : \quad Unif([0,1])$$

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Evaluating critical smoothing factor

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Evaluating critical smoothing factor

Nonparametric, kernel *PDF* estimate:

$$\hat{f}_n\left(M \mid \{M_i\}, h\right) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{M - M_i}{h}\right)$$

$$K(\xi) = (2\pi)^{-0.5}\exp\left(-\xi^2/2\right) \quad - \quad kernel$$

$$M_i, i = 1,..,n \quad - \quad data$$

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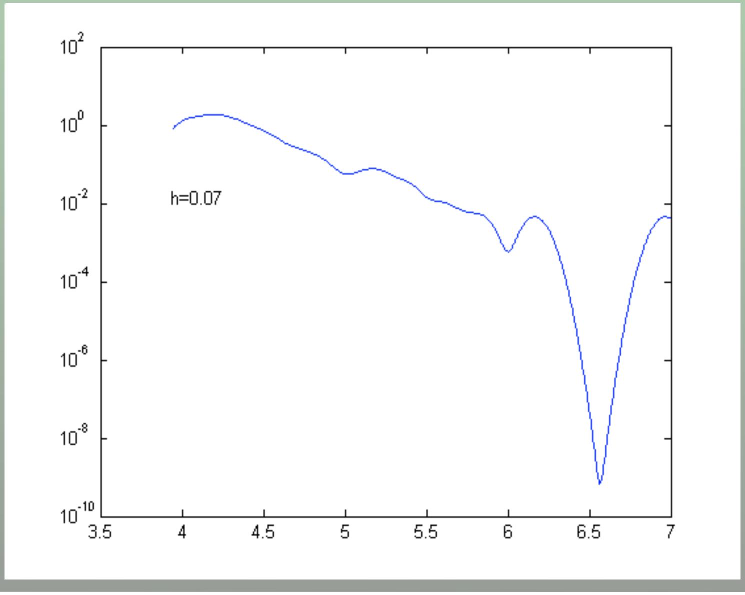

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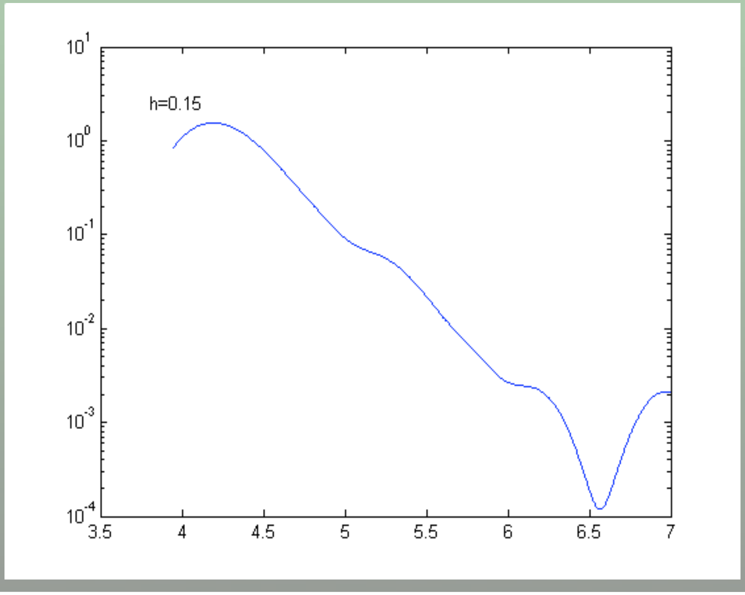

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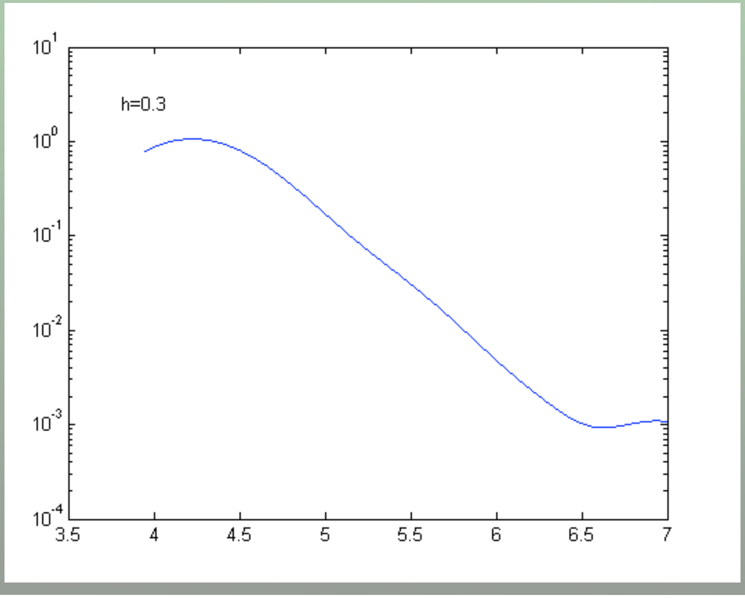

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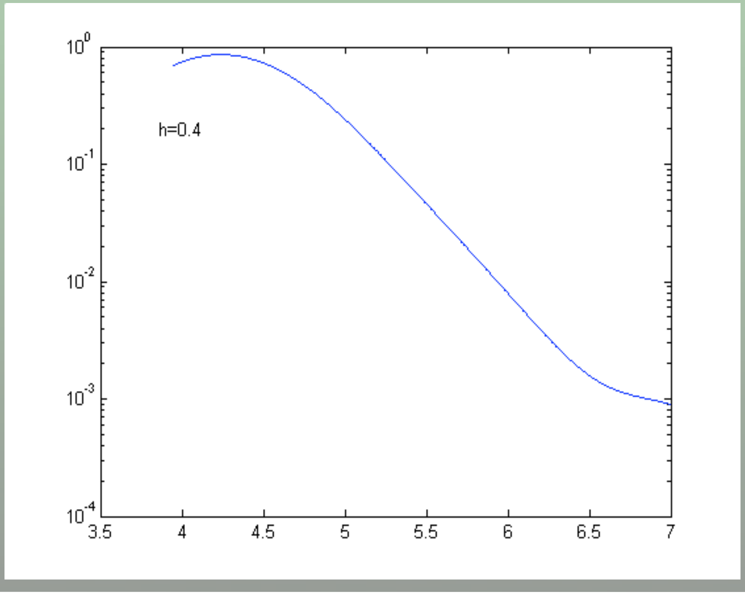

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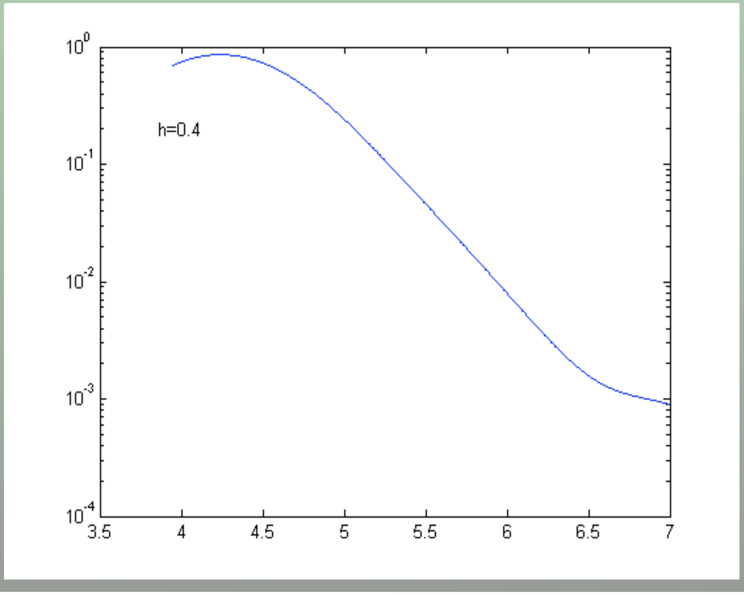

Critical smoothing factor h_{cr}:

$$\hat{f}_n\left(M \mid \{M_i\}, h\right) \quad - \quad \begin{cases} unimodal\ \ for\ h \ge h_{cr} \\ multimodal\ \ for\ h < h_{cr} \end{cases}$$

TESTING FOR MULTIMODALITY

Smooth bootstrapping

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Standard bootstrap: $\{M_i\}_{i=1,..n}$ ® $\{M_i^{'}\}_{i=1,..,n}$

TESTING FOR MULTIMODALITY

Smooth bootstrapping

Smooth bootstrapping = Sampling from $\hat{f}_n\left(M \mid \{M_i\}, h_{cr}\right)$

Standard bootstrap: $\{M_i\}_{i=1,..n} \circledR \{M_i^{'}\}_{=1,..,n}$

Smooth bootstrap:

$$\{M_i^{Silv}\}_{=1,..,n} : M_i^{Silv} = M_i^{'} + h_{cr}\varepsilon_i$$

Silverman (1986):

$$\{M_i^{Efr}\}_{=1,..,n} : M_i^{Efr} = \overline{M}^{Silv} + \frac{M_i^{Silv} - \overline{M}^{Silv}}{\sqrt{1 + h_{cr}^{2} / \hat{\sigma}^{2}}}$$

Efron, Tibshirani (1998):

$$\varepsilon : \quad Norm(0,1) \qquad \hat{\sigma}^2 = \mathrm{samplevar}(M_i)$$

TESTING FOR MULTIMODALITY
Estimating H_0 significance

R bootstrap samples

$$\hat{P}_{Silv}(H_0) = \frac{the\ number\ of\ unimodal\ \hat{f}_n\left(M \mid \{M_i^{Silv}\} h_{cr}\right)}{R}$$

$$\hat{P}_{Efr}(H_0) = \frac{the\ number\ of\ unimodal\ \hat{f}_n\left(M \mid \{M_i^{Efr}\} h_{cr}\right)}{R}$$

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TEST CALIBRATION

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Test result $\hat{P}(H_0) = p^*$

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If H_0 were true how many samples like $\{M_i\}$ would result in $\hat{P}(H_0) \leq p^*$?

TEST CALIBRATION

Fitting G-R distribution model

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Fitting G-R distribution model

$$\{M_i\} \Rightarrow \hat{F}_{GR}(M) = \begin{cases} 0 & M < M_c \\ \dfrac{1 - \exp\left[-\hat{\beta}\left(M - M_c\right)\right]}{1 - \exp\left[-\hat{\beta}\left(\hat{M}_{\max} - M_c\right)\right]} & M_c \le M \le \hat{M}_{\max} \\ 1 & M > \hat{M}_{\max} \end{cases}$$

$$\hat{M}_{\max} = M_{\max}^{obs} + \int_{M_c}^{\hat{M}_{\max}} [\hat{F}_{GR}(M)]^n \, dM \qquad M_{\max}^{obs} = \max_i (M_i)$$

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Fitting G-R distribution model

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$\hat{F}_{GR}(M)$: - supports H_0

- could underlay $\{M_i\}$

TEST CALIBRATION

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$$\hat{F}_{GR}(M) \circledR \left\lfloor \{ M_i^{GR} \}_{i=1,..,n} \right\rfloor_{k=1,..,L}$$

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$$L \; values \; \hat{P}(H_0) ® \; \forall p^* \; \nu = \frac{No \; of \; \hat{P}(H_0) < p^*}{L}$$

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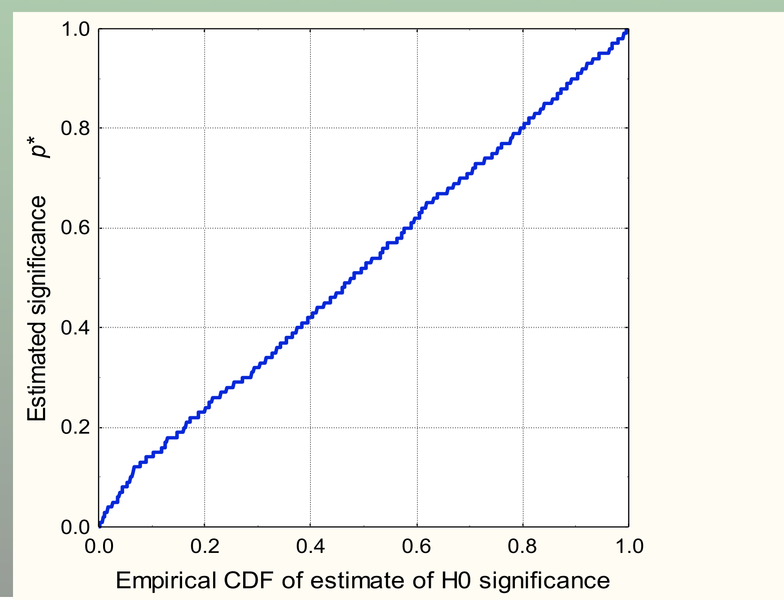
p^* vs ν : calibration curve



SELECTED RESULTS

- REGIONAL CATALOGS

Southern-California earthquakes (*SCSN Format EQ Catalog*)

983 EQ-s from 1.07.1944-1.03.1990 $M \geq 4.0$

Area definition: Nordquist (1964)

Sample selection: Knopoff (2000)

Prob{H_0^1: unimodality} = 0.102

Prob{H_0^2: one bump} = 0.091

SELECTED RESULTS

- REGIONAL CATALOGS

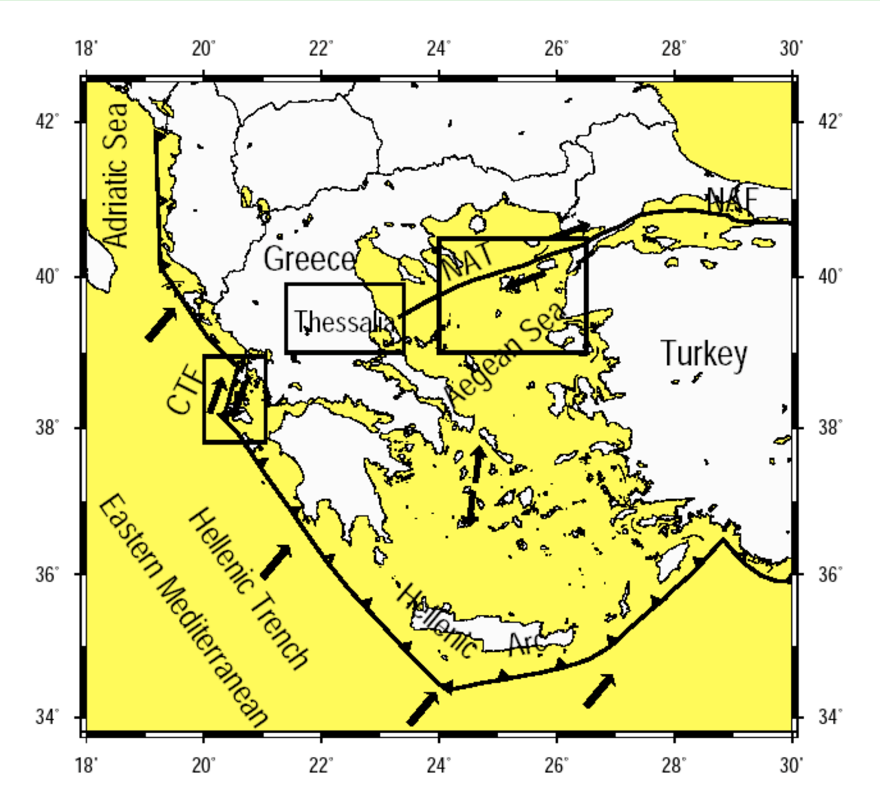
Northern-California earthquakes: Northern California Earthquake Catalog and Phase Data (*Northern California Seismic Network, U.S. Geological Survey, Menlo Park; Berkeley Seismological Laboratory, University of California, Berkeley*)

603 EQ-s from 1.01.1968-31.12.2006 $M \geq 4.0$ lat>38^o

Prob{H_0^1: unimodality} = 0. 042
Prob{H_0^2: one bump} = 0.074

SELECTED RESULTS

- REGIONAL CATALOGS - GREECE



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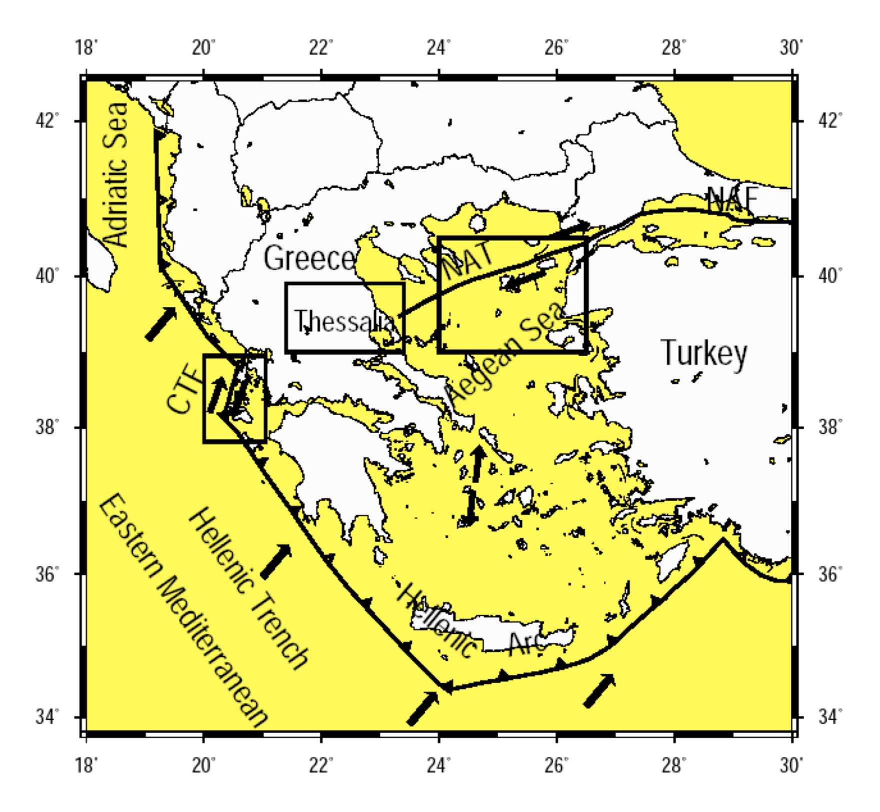

Central Ionian Islands earthquakes:

1256 EQ-s from 1981-2001 $M \geq 4.0$

Prob{H_0^1: unimodality} = 0.069

Prob{H_0^2: one bump} = 0.080

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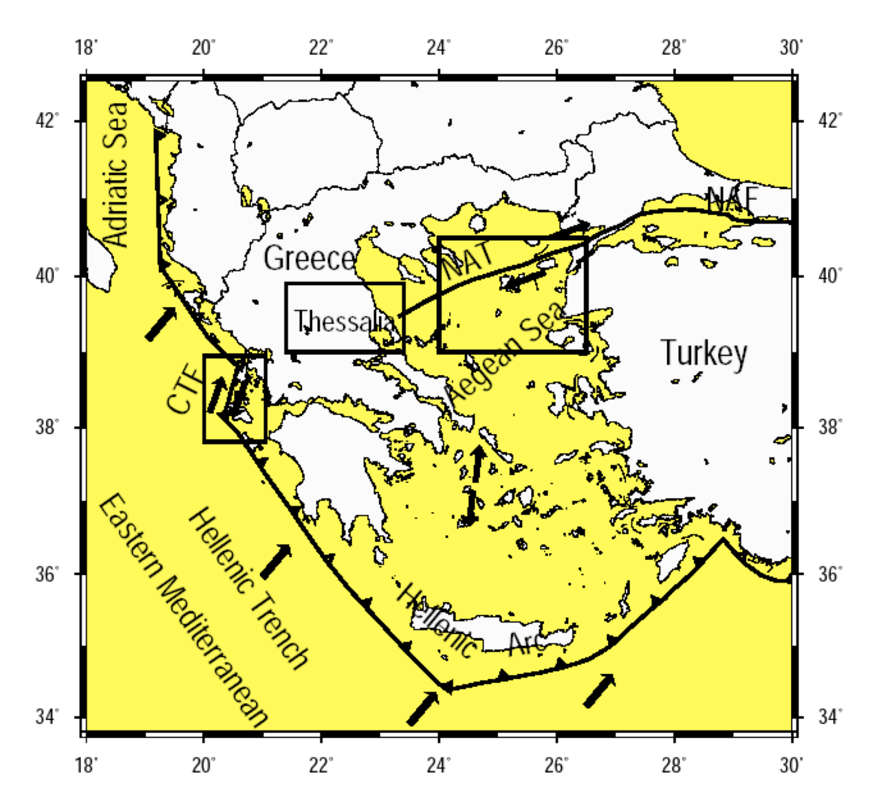

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1256 EQ-s from 1981-2001 $M \geq 4.0$

Prob{H_0^1: unimodality} = 0.069
Prob{H_0^2: one bump} = 0.080

Northern Aegean earthquakes:
744 EQ-s from 1981-2001 $M \geq 4.0$

Prob{H_0^1: unimodality} = 0.049
Prob{H_0^2: one bump} = 0.076

SELECTED RESULTS

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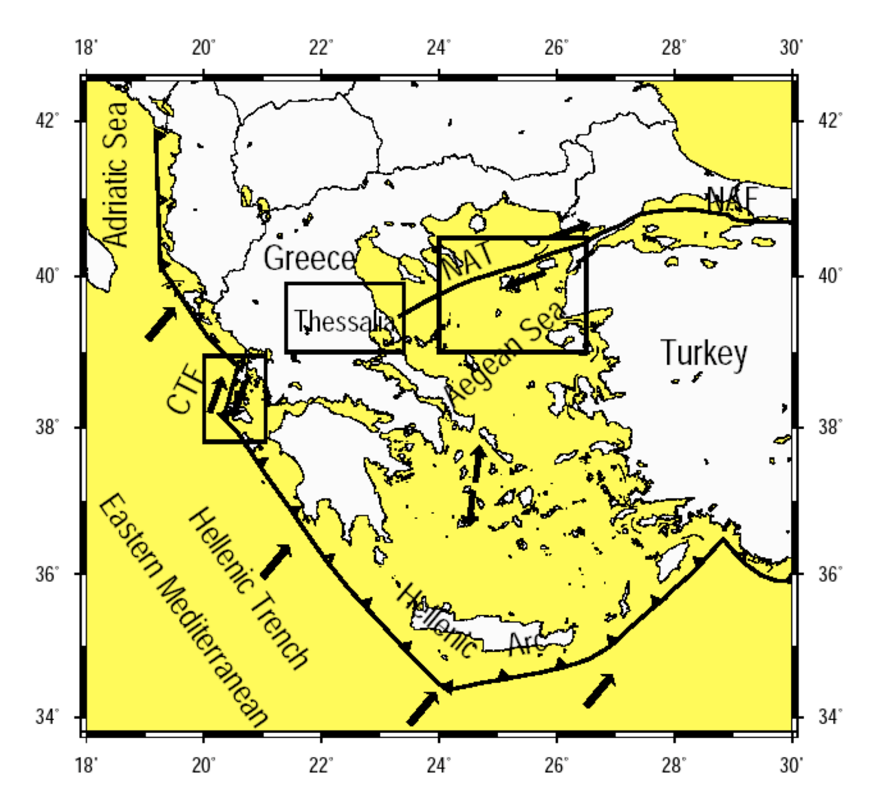

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Prob$\{H_0^2$: one bump$\}$ = 0.080

Northern Aegean earthquakes:
744 EQ-s from 1981-2001 $M \geq 4.0$

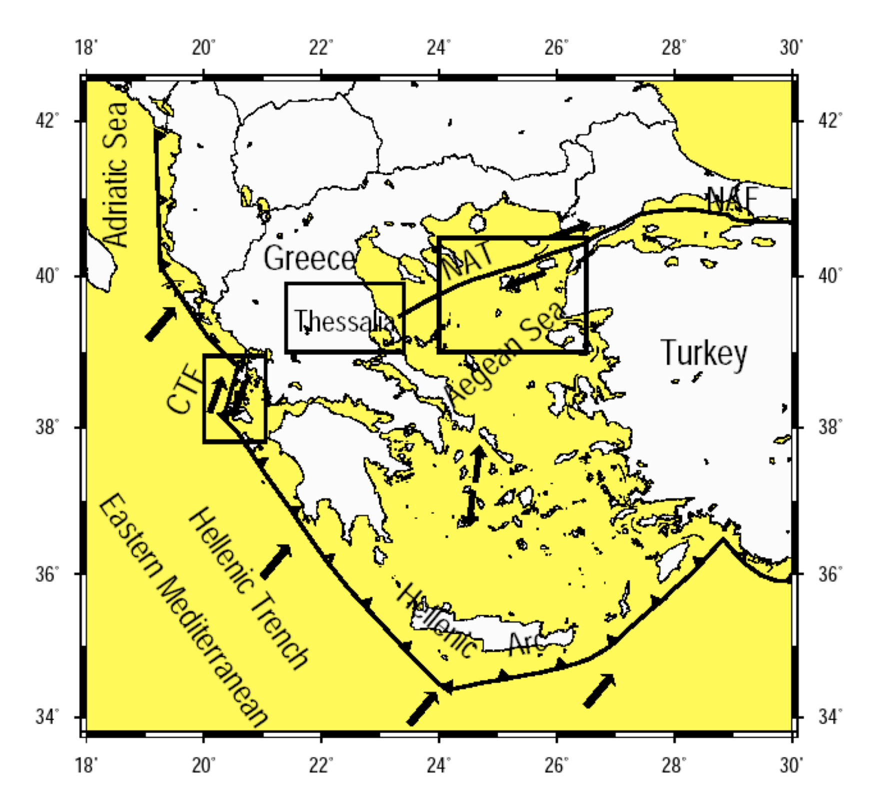
Prob$\{H_0^1$: unimodality$\}$ = 0.049
Prob$\{H_0^2$: one bump$\}$ = 0.076

Thessalia earthquakes:
104 EQ-s from 1981-2001 $M \geq 4.0$

Prob$\{H_0^1$: unimodality$\}$ = 0.51
Prob$\{H_0^2$: one bump$\}$ = 0.23

SELECTED RESULTS

- REGIONAL CATALOGS – GREECE



Central Ionian Islands earthquakes with aftershocks removed (*Reasenberg, 1985*):

595 EQ-s from 1981-2001 $M \geq 4.0$

Prob$\{H_0^1$: unimodality$\} = 0.050$

Prob$\{H_0^2$: one bump$\} = 0.062$

SELECTED RESULTS

- WORLDWIDE CATALOGS

Large ($M \geq 7.0$), shallow ($h \leq 70$km), worldwide earthquakes: combined Pacheco-Sykes catalog (698 EQ-s, *Pacheco and Sykes, 1992*) and Harvard CMTS catalogs:

821 EQ-s from 1900-2002

Prob$\{H_0^1$: unimodality$\}$ = 0.362
Prob$\{H_0^2$: one bump$\}$ = 0.096

SELECTED RESULTS

- WORLDWIDE CATALOGS

Worldwide earthquakes: Harvard CMTS catalog

1825 EQ-s from 1.01.1977-31.12.2004 $Mw \geq 6.0$

Prob$\{H_0^1$: unimodality$\} = 0.162$

Prob$\{H_0^2$: one bump$\} < 9 \times 10^{-4}$

CONCLUSIONS

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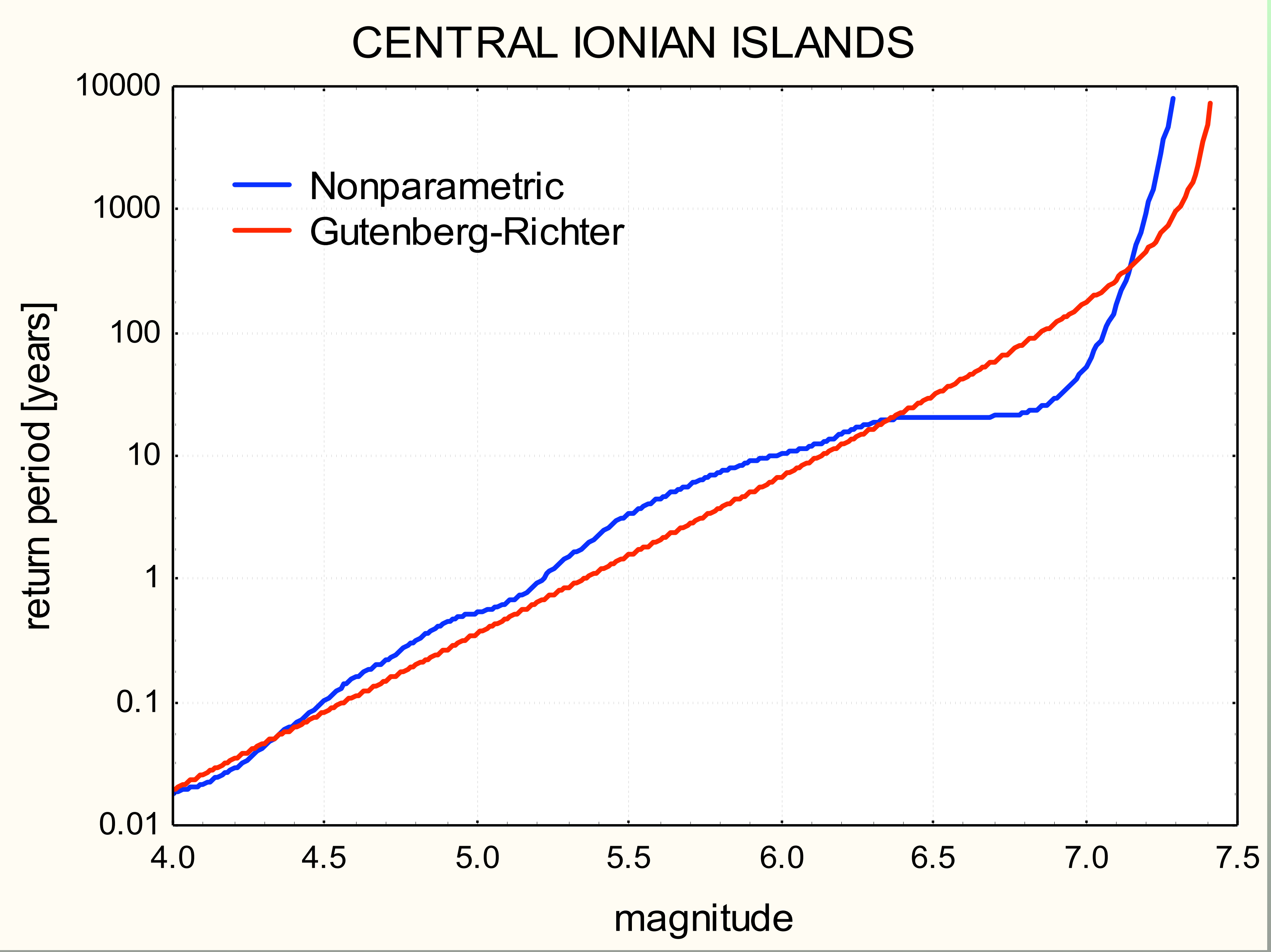
CONCLUSIONS

- *The smoothed bootstrap test for multimodality makes it possible to investigate the complexity of earthquake magnitude distribution without making any specific assumptions on the distribution model.*
- *The test evidence that the earthquake magnitude distribution frequently neither follows the Gutenberg-Richter law nor is smoothly non-linear but it is complex. Regarding magnitudes, earthquake populations are often not homogeneous. Traces of the complexity are more distinct for the regional than the worldwide data.*
- *The complexity of magnitude distribution has also important implications for probabilistic seismic hazard assessment. When the actual magnitude distribution is complex and nonlinear features occur in a large magnitude region, the use of the presently known magnitude distribution models may yield unacceptable inaccuracy of the hazard estimates.*


CENTRAL IONIAN ISLANDS
Nonparametric
Gutenberg-Richter
return period [years]
10000
1000
100
10
1
0.1
0.01
4.0
4.5
5.0
5.5
6.0
6.5
7.0
7.5
magnitude

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